

**Mathematics Ext 1
Preliminary & HSC Course
Outline**

This book belongs to:

Information for Students

This booklet is an important document. It explains the outcomes of the course you are studying and is a valuable resource for when you are studying. The performance bands included explain to you what you need to do to demonstrate your knowledge, understanding and skills.

If you miss lessons **it is your responsibility** to find out if any information about assessment tasks was given out during the period of absence in addition to catching up on any missed work. In cases of prolonged absence you should request that school work be sent home for you to complete.

If you don't understand what is required of you in any assessment task **it is your responsibility** to seek help from your class teacher or the Head Teacher.

You will need to attend each lesson and complete all class work. The Board of Studies may refuse to grant a Higher School Certificate to a student whose attendance or application to work has been unsatisfactory.

It is your responsibility to carefully read and understand this information and ask for any clarification if you do not understand.

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A Glossary of Key Words

Syllabus outcomes, objectives, performance bands and examination questions have key words that state what students are expected to be able to do. A glossary of key words has been developed to help provide a common language and consistent meaning in the Higher School Certificate documents.

Account	Account for: state reasons for, report on. Give an account of: narrate a series of events or transactions
Analyse	Identify components and the relationship between them; draw out and relate implications
Apply	Use, utilise, employ in a particular situation
Appreciate	Make a judgment about the value of
Assess	Make a judgment of value, quality, outcomes, results or size
Calculate	Ascertain/determine from given facts, figures or information
Clarify	Make clear or plain
Classify	Arrange or include in classes/categories
Compare	Show how things are similar or different
Construct	Make: build; put together items or arguments
Contrast	Show how things are different or opposite
Critically (analyze/evaluate)	Add a degree or level of accuracy, depth, knowledge understanding, logic, questioning, reflection and quality to (analysis/evaluation)
Deduce	Draw conclusions
Define	State meaning and identify essential qualities
Demonstrate	Show by example
Describe	Provide characteristics and features
Discuss	Identify issues and provide points for and/or against
Distinguish	Recognise or note/indicate as being distinct or different from; to note differences between
Evaluate	Make a judgment based on criteria; determine the value of
Examine	Inquire into
Explain	Relate cause and effect; make the relationships between things evident; provide why and/or how
Extract	Choose relevant and/or appropriate details
Extrapolate	Infer from what is known
Identify	Recognise and name
Interpret	Draw meaning from
Investigate	Plan, inquire into and draw conclusions about
Justify	Support an argument or conclusion
Outline	Sketch in general terms; indicate the main features of
Predict	Suggest what may happen based on available information
Propose	Put forward (for example a point of view, idea, argument, suggestion) for consideration or action
Recall	Present remembered ideas, facts or experiences
Recommend	Provide reasons in favour
Recount	Retell a series of events
Summarise	Express, concisely, the relevant details
Synthesise	Putting together various elements to make a whole

MATHEMATICS EXTENSION 1

AIMS:

- to give an understanding of important mathematical ideas such as variable, function, limit etc, and to introduce students to mathematical techniques which are relevant to the real world.
- to understand the need to prove results, to appreciate the role of deductive reasoning in establishing such proofs, and to develop the ability to construct such proofs.
- to enhance those mathematical skills required for further studies in mathematics, the physical sciences and the technological sciences.

Mathematics Extension 1 Outcomes

Objectives	Preliminary Outcomes	HSC Outcomes
Students will develop:	A student:	A student:
appreciation of the scope, usefulness, beauty and elegance of mathematics	PE1 appreciates the role of mathematics in the solution of practical problems	HE1 appreciates interrelationships between ideas drawn from different areas of mathematics
the ability to reason in a broad range of mathematical contexts	PE2 uses multi-step deductive reasoning in a variety of the context	HE2 uses inductive reasoning in the construction of contexts proofs
skills in applying mathematical techniques to the solution of practical problems	PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations	HE3 uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay
understanding of the key concepts of calculus and the ability to differentiate and integrate a range of functions	PE4 uses the parametric representation together with differentiation to identify geometric properties of parabolas	HE4 uses the relationship between functions, inverse functions and their derivatives

	<p>PE5 determines derivatives which require the application of more than one rule of differentiation</p>	<p>HE5 applies the chain rule to problems including those involving velocity and acceleration as functions of displacement</p> <p>HE6 determines integrals by reduction to a standard form through a given substitution</p>
<p>the ability to interpret and communicate mathematics in a variety of forms</p>	<p>PE6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations</p>	<p>HE7 Evaluates mathematical solutions to problems and communicates them in an appropriate form</p>

MATHEMATICS EXTENSION 1

The Mathematics Extension 1 syllabus has been divided into a Preliminary Course and HSC course as follows:

Preliminary Course	HSC Course
Other inequalities (1.4 E)	Methods of integration (1 1.5)
Circle geometry (2.6 - 2.10)	Primitive of $\sin^2 x$ and $\cos^2 x$ (13.6 E)
Further trigonometry (sums and differences, t formulae, identities and equations) (5.6 - 5.9)	Equation $\frac{dN}{dt} = k(N-P)$ (14.2E)
Angles between 2 lines (6.6)	Velocity and acceleration as a function of x (14.3 E)
Internal and external division of lines into given ratios (6.7 E)	Projectile motion (14.3 E)
Parametric representation (9.6)	Simple Harmonic motion (14.4)
Permutations and combinations (18.1)	Inverse functions and inverse trigonometric functions (15.1 - 15.5)
Polynomials (16.1 -16.3)	Binomial theorem (17.1 - 17.3)
Induction (17.4)	Further probability (18.2)
Harder application of the Preliminary 2 Unit course	Iterative methods for numerical estimation of the roots of polynomial equation (16.4)
	Harder application for HSC 2 Unit topics

Preliminary Course

1 OTHER INEQUALITIES

Solve inequalities such as $\frac{x^2 - 1}{x} > 0$, $\frac{2t + 1}{t - 2} > 1$

2 FURTHER TRIGONOMETRY

- a Trigonometric functions of sums and differences of angles
 (i) Formal proof that $\cos(A-B) = \cos A \cos B + \sin A \sin B$
 (ii) Use of the above to derive the corresponding identities for $\cos(A+B)$, $\sin(A-B)$, $\sin(A+B)$, $\tan(A-B)$ and $\tan(A+B)$
 (iii) Extension of the above to derive identities of $\sin 2A$, $\cos 2A$, $\tan 2A$, $\sin 3A$, $\cos 3A$ and $\tan 3A$
- b
$$t = \tan\left(\frac{A}{2}\right)$$

 Expression of $\sin a$, $\cos a$ and $\tan a$ in terms of t
- c Solving trigonometric equations. General solution
 d Equations of the form $a \cos A + b \sin A = c$
 e Differentiation
 f Integration and applications to area and volume

3 ANGLES BETWEEN TWO LINES

The formula should be derived, and use made of it in solving problems

4 INTERNAL AND EXTERNAL DIVISION OF LINES INTO GIVEN RATIOS

The coordinates of the points dividing a given interval in the ratio $m : n$, internally and externally, should be derived

5 PARAMETRIC REPRESENTATION

- a Parametric equation $x = 2at$, $y = at^2$
 b Equations of the tangent and normal at the point 't' and at (x_1, y_1)
 c Equation of the chord of contact of the tangent from an external point
 d Prove the following analytically;
 (i) tangent to a parabola at a given point is equally inclined to the axis and the focal chord through the point i.e. principle of the parabolic reflector
 (ii) All tangents at the extremities of a focal chord intersect at right angles on the directrix
 e Simple locus problems, one and two variables

6 POLYNOMIALS

- a The general polynomial $P(x) = p_0x^n + p_1x^{n-1} + \dots + p_n$, the polynomial function $y = p(x)$ and the polynomial equation $P(x) = 0$
 b Meaning and use of the terms degree, coefficient, leading term, constant term, monic polynomial, zeros, roots
 c Sketching polynomials without the aid of calculus. Include;
 (i) Graphical significance of double and treble roots
 (ii) for very large x , $P(x) = p_nx^n$
 (iii) a polynomial of odd degree always has at least 1 real zero
 d Division transformation with polynomials (linear and quadratic divisors)
 e Remainder and Factor Theorems. Deductions from the factor theorem
 f Equality of polynomials
 g Relationship between roots and coefficients of polynomial equations

7 INDUCTION

- a The principal of mathematical induction
 b Application: geometrical, inequalities, series, etc

8 CIRCLE GEOMETRY

- a Definition of circle, centre, radius, diameter, arc, sector, segment, chord, tangent, concyclic points, cyclic quadrilateral, angle subtended by an arc or chord at the centre and at the circumference, and of an arc subtended by an angle.
 b Two circles touch if they have a common tangent at the point of contact
 c Equal arc on circles of equal radii subtend equal angles at the centre. Converse.
 ~ Prove the following;
 d Equal angles at the centre stand on equal chords. Converse
 e The angle at the centre is twice the angle at circumference subtended by same arc.
 f Tangent to circle is perpendicular to radius drawn to point of contact. Converse.
 g Perpendicular from centre of a circle to a chord bisects the chord
 h Line from the centre of a circle to midpoint of a chord is perpendicular to chord
 i Equal chords in equal circles are equidistant from the centres
 j Chords in a circle which are equidistant from the centre are equal
 k Any 3 non-collinear points lie on a unique circle, whose centre is the point of concurrency of the perpendicular bisector of the intervals joining the points.
 l Angles in the same segment are equal

- m The angle in a semi-circle is a right angle
- n Opposite angles in a cyclic quadrilateral are supplementary
- o The exterior angle at a vertex of a cyclic quadrilateral equals the interior opposite angle
- p If the opposite angles in a quadrilateral are supplementary then the quadrilateral is cyclic (Also test for 4 points to be concyclic)
- q If an interval subtends equal angles at 2 points on the same side of it then the end points of the interval and two points are concyclic
- r The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment
- s Tangents to a circle from an external point are equal
- t The square of the length of the tangent from an external point is equal to the product of the intercepts of the secant passing through this point
- u When circles touch, the line of centres passes through the point of contact

9 PERMUTATIONS AND COMBINATIONS

- a Systematic enumeration in a finite sample space
- b Basic counting theory
- c Ordered selections (permutations)
- d Unordered selections (combinations)
- e Meaning of $n!$, ${}^n P_r$, ${}^n C_r$, also written $\binom{n}{r}$
- f Gambling and games of chance
- g Arrangement in a circle

HSC Course

A PRIMITIVE OF $\sin^2 x$ AND $\cos^2 x$

B THE INVERSE TRIGONOMETRIC FUNCTIONS

- a Revision of notion of inverse functions
- b $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$
- c Definition of properties of the inverse trigonometric functions. Include;
 - (i) domain, range
 - (ii) symmetry properties eg $\sin^{-1}(-x) = -\sin^{-1}(x)$
 - (iii) $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$
- d Graphs of $\sin^{-1}(x)$, $\cos^{-1}(x)$ and $\tan^{-1}(x)$
- e The derivatives of $\sin^{-1}\left(\frac{x}{a}\right)$, $\cos^{-1}\left(\frac{x}{a}\right)$ and $\tan^{-1}\left(\frac{x}{a}\right)$ and applications
- f Corresponding integrations and applications

C EQUATION $\frac{dN}{dt} = k(N - P)$

- a One solution of the equation is $N=P$
- b Direct substitution shows a solution in the form $N = P + Ae^{kt}$
- c Numerical examples determining A and/or k from given initial conditions
- d Discuss cases where $k < 0$ and $k > 0$

D VELOCITY AND ACCELERATION AS A FUNCTION OF X

Derive and apply; $\frac{d^2x}{dt^2} = \frac{d}{dt} v(x) = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$

E PROJECTILE MOTION

- a Derive equations of motion of a particle projected vertically upwards
- b Two-dimensional motion of a projectile with initial conditions leads to $x = vt \cos A$,

$$y = vt \sin A - \frac{1}{2} gt^2$$

This gives a parametric representation of the "flight parabola"

- c Cartesian equation is $y = x \tan A - \frac{1}{2} gx^2 / (v^2 \cos^2 A)$
- d Derive range for a projectile fired on a horizontal plane
- e Maximum range on horizontal plan is $\frac{v^2}{g}$ when $A = 45^\circ$

F SIMPLE HARMONIC MOTION

- a From $v = \dot{x} = -an \sin(nt + A)$ to $a = \ddot{x} = -an^2 \cos(nt + A) = -an^2$
- b Graphs of x , \dot{x} and \ddot{x} as functions of t should be sketched
- c Relationships between zero, min and max, values of the 3 quantities
- d Physical significance of the parameters a , n and A
- e Define amplitude, frequency, period and phase
- f Description of differential equation of the motion
- g Use of $v^2 + n^2 x^2 = a^2 n^2$
- h Extension to $x = b + a \cos(nt + A)$ and $\ddot{x} = -n^2(x - b)$. SHM about $x = b$

G ITERATIVE METHODS FOR NUMERICAL ESTIMATION OF THE ROOTS OF A POLYNOMIAL EQUATION

- a Halving the interval
- b Newton's method

H	METHODS OF INTEGRATION
a	Harder Mathematics course integration
b	Reduction to standard form by very simple substitutions

I	BINOMIAL THEOREM
a	Expansion of $(1+x)^n$ and $(a+b)^n$ for $n = 1, 2, 3, 4, 5$
b	Use of Pascal's Triangle in expansions of $(a+b)^n$ for positive integers
c	${}^n C_r$ defined as the coefficient x^n in the binomial expansion of $(a+b)^n$
d	Proof of the Pascal's Triangle relationship ${}^{n+1} C_r = {}^n C_{r-1} + {}^n C_r$
e	Hypothesis that ${}^n C_r = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$ as a result of the pattern revealed in successive differentiation of the identity
	$(1+x)^n = \sum_{r=0}^{r=n} {}^n C_r x^r$ <p>after letting $x = 0$, each time equating corresponding powers</p>
f	${}^n C_r = \frac{n!}{r!(n-r)!}$. Proof of both formulae for ${}^n C_r$ by mathematical induction
g	General term of binomial expansion of $(a+b)^n$, $T_{r+1} = {}^n C_r a^{n-r} b^r$
h	The greatest coefficient and greatest term in an expansion
l	Relations between the binomial coefficients
j	Finite series and further properties of binomial coefficients
k	Miscellaneous exercise on expansions, coefficients, specific terms, etc

J	PROBABILITY
a	Historical background
b	Random experiments, equally likely outcomes, probability of a given result
c	Finite sample space - simple, composite and mutually exclusive event, Venn diagrams
d	Diagrammatic representation of sample spaces. Counting techniques
e	$P(E) = \frac{n(E)}{n(S)}$, $0 < P(E) < 1$ Complementary results defined
f	Addition theory or theorem of total probability
g	Product theorem
h	Dependent events - tree diagram (2 or 3 stage experiments)
i	Independent events - product rule, tree diagrams

K	FURTHER PROBABILITY
a	Review of the Mathematics course probability
b	Applications of permutations and combinations to probability
c	Binomial probabilities and the binomial distribution
d	Numerous miscellaneous exercises including gambling situations
e	Expected values

L	GENERAL REVISION
a	Topic by topic
b	Specimen papers
c	Trial HSC papers
d	HSC papers

Marking Criteria

At the completion of your HSC course you will receive a Certificate of Achievement that indicates your level of achievement in each of your courses. Your final mark for each subject will place you in one of 6 bands (similar to your School Certificate), 6 being the highest and 2 being the lowest with only the minimum standard being achieved. Band 1 will be below the minimum standard needed to successfully complete the course.

To achieve a result in Bands 5 or 6 you need to consistently present solutions as described in the “Best Solutions” below. To achieve a result in bands 3 or 4 you need to consistently present solutions as described in the “Average Solutions” below. Presented solutions as described in the “Minimum Standard Solutions” below may give you a result in Band 2.

Your final mark is an average of your school assessment and your HSC examination mark. By the time you have completed the Preliminary Course you should have developed sufficient knowledge, skills and work habits to be able to consistently set out your solutions as described in “The Best Solutions” section below. As the school assessment is continuous throughout the HSC you will need to apply these points to all your work for the HSC course.

The Best Solutions should:

- Use a wide variety of problem solving strategies, successfully applying mathematical skills and processes to the most appropriate method.
- Clearly show understanding of the questions intent.
- Correctly use the language of mathematics including symbols, abbreviations, notation and conventions.
- Use graphs and diagrams to show understanding of the problem and as an aid in finding the solution.
- Demonstrate a good understanding of all the mathematical concepts involved.
- Recognise and fully explain each step involved in the solution in a clear logical sequence.
- Draw diagrams and graphs that are clearly labelled, reasonably sized and well executed using correct geometrical instruments (ruler, set square, pair of compasses).
- Give answers that are mathematically correct, expressed in the simplest form and with the correct units.
- State any formula used, clearly show the substitution and then evaluate.
- Clearly show the final answer, including writing the answer in a sentence when appropriate.
- Avoid using rounded values during the course of calculations and write down the full calculator answer before giving a rounded off answer.

- Leave incorrect and rough working still legible by only using a single line to cross out unwanted working in a solution.
- Show working spread out neatly, working down the page, not across, avoid using columns, ensure numbered parts are clearly indicated.
- Avoid irrelevant algebra and number crunching, or proving given information or giving essay answers when a sentence (or two) is enough.

The Average Solution should:

- Use a variety of problem solving strategies, applying mathematical skills and processes to the method used with only minor errors.
- Clearly show understanding of the questions intent.
- Correctly use the language of mathematics including symbols, abbreviations, notation and conventions in most parts of the solution.
- Use graphs and diagrams to show understanding of the problem and as an aid in finding the solution.
- Generally demonstrate understanding of the mathematical concepts involved.
- Explain each step involved in the solution in a clear logical sequence.
- Draw diagrams and graphs that are clearly labelled, reasonably sized and well executed using correct geometrical instruments (ruler, set square, pair of compasses).
- Generally give answers that are mathematically correct, expressed in the simplest form and with the correct units.
- State any formula used, clearly show the substitution and then evaluate.
- Clearly show the final answer, including writing the answer in a sentence when appropriate.
- Avoid using rounded values during the course of calculations and write down the full calculator answer before giving a rounded off answer.
- Leave incorrect and rough working still legible, using only a single line to cross out unwanted working in a solution.
- Show working spread out neatly, working down the page, not across, avoid using columns, ensure numbered parts are clearly indicated.
- Generally avoid irrelevant algebra and number crunching, or proving given information or giving essay answers when a sentence (or two) is enough.

The Minimum Standard Solutions should:

- Apply mathematical skills and processes to the method used with only minor errors.
- Correctly use some of the language of mathematics including symbols, abbreviations, notation and conventions.
- Use graphs and diagrams as an aid in finding the solution.
- Demonstrate some understanding of the mathematical concepts involved.
- Explain steps involved in the solution.

- Draw diagrams and graphs that are clearly labelled, reasonably sized and well executed using correct geometrical instruments (ruler, set square, pair of compasses).
- Generally give answers that are mathematically correct with correct units.
- State any formula used, clearly show the substitution and then evaluate.
- Clearly show the final answer, including writing the answer in a sentence when appropriate.
- Avoid using rounded values during the course of calculations and write down the full calculator answer before giving a rounded off answer.
- Leave incorrect and rough working still legible, using a single line to cross out unwanted working in a solution.
- Show working spread out neatly, working down the page, not across, avoid using columns, ensure numbered parts are clearly indicated.

DRAFT PERFORMANCE BANDS

MATHEMATICS EXTENSION 1

The typical performance in this band:

Band E4	<ul style="list-style-type: none"> <input type="checkbox"/> Exhibits extensive knowledge and skills appropriate to the Mathematics and Mathematics Extension 1 courses <input type="checkbox"/> Synthesises mathematical techniques, results and ideas creatively across the Mathematics and Mathematics Extension 1 courses to solve difficult problems <input type="checkbox"/> Uses sophisticated multi-step mathematical reasoning <input type="checkbox"/> Interprets, explains, justifies and evaluates solutions to problems <input type="checkbox"/> Translates efficiently between practical problems and their mathematical model <input type="checkbox"/> Communicates complex ideas and arguments effectively using appropriate mathematical language, notation, diagrams and graphs
Band E3	<ul style="list-style-type: none"> <input type="checkbox"/> Exhibits knowledge and skills appropriate to the Mathematics and Mathematics Extension 1 courses <input type="checkbox"/> Synthesises mathematical techniques, results and ideas from across the Mathematics and Mathematics Extension 1 courses to solve problems in areas such as geometry, calculus and probability <input type="checkbox"/> Uses multi-step mathematical reasoning such as mathematical induction <input type="checkbox"/> Translates between practical problems and their mathematical model in areas such as projectile motion <input type="checkbox"/> Communicates effectively using appropriate mathematical language, notation, diagrams and graphs
Band E2	<ul style="list-style-type: none"> <input type="checkbox"/> Exhibits knowledge of the techniques of the Mathematics and Mathematics Extension 1 courses <input type="checkbox"/> Uses techniques of integration from the Mathematics Extension 1 course such as integration by substitution <input type="checkbox"/> Uses logical reasoning in numerical contexts such as problems in algebra and geometry <input type="checkbox"/> Applies calculus to solve practical problems <input type="checkbox"/> Communicates using mathematical language, notation, diagrams and graphs
Band E1	