Mathematics
Preliminary & HSC Course Outline

Information for Students

This booklet is an important document. It explains the outcomes of the course you are studying and is a valuable resource for when you are studying. The performance bands included explain to you what you need to do to demonstrate your knowledge, understanding and skills.

If you miss lessons **it is your responsibility** to find out if any information about assessment tasks was given out during the period of absence in addition to catching up on any missed work. In cases of prolonged absence you should request that school work be sent home for you to complete.

If you don't understand what is required of you in any assessment task **it is your responsibility** to seek help from your class teacher or the Head Teacher.

You will need to attend each lesson and complete all class work. The Board of Studies may refuse to grant a Higher School Certificate to a student whose attendance or application to work has been unsatisfactory.

It is your responsibility to carefully read and understand this information and ask for any clarification if you do not understand.

Contents

A Glossary of Key Words

Syllabus outcomes, objectives, performance bands and examination questions have key works that state what students are expected to be able to do. A glossary of key words has been developed to help provide a common language and consistent meaning in the Higher School Certificate documents.

Account	Account for: state reasons for, report on. Give an account of: narrate a series of events or transactions
Analyse	Identify components and the relationship between them; draw out and relate implications
Apply	Use, utilise, employ in a particular situation
Appreciate	Make a judgment about the value of
Assess	Make a judgment of value, quality, outcomes, results or size
Calculate	Ascertain/determine from given facts, figures or information
Clarify	Make clear or plain
Classify	Arrange or include in classes/categories
Compare	Show how things are similar or different
Construct	Make: build; put together items or arguments
Contrast	Show how things are different or opposite
Critically (analyze/evaluate)	Add a degree or level of accuracy, depth, knowledge understanding, logic, questioning, reflection and quality to (analysis/evaluation)
Deduce	Draw conclusions
Define	State meaning and identify essential qualities
Demonstrate	Show by example
Describe	Provide characteristics and features
Discuss	Identify issues and provide points for and/or against
Distinguish	Recognise or note/indicate as being distinct or different from; to note differences between
Evaluate	Make a judgment based on criteria; determine the value of
Examine	Inquire into
Explain	Relate cause and effect; make the relationships between things evident; provide why and/or how
Extract	Choose relevant and/or appropriate details
Extrapolate	Infer from what is known
Identify	Recognise and name
Interpret	Draw meaning from
Investigate	Plan, inquire into and draw conclusions about
Justify	Support an argument or conclusion
Outline	Sketch in general terms; indicate the main features of
Predict	Suggest what may happen based on available information
Propose	Put forward (for example a point of view, idea, argument, suggestion) for consideration or action
Recall	Present remembered ideas, facts or experiences
Recommend	Provide reasons in favour
Recount	Retell a series of events
Summarise	Express, concisely, the relevant details
Synthesise	Putting together various elements to make a whole

MATHEMATICS

AIMS:

- to give an understanding of important mathematical ideas such as variable, function , limit etc, and to introduce students to mathematical techniques which are relevant to the real world
- to understand the need to prove results, to appreciate the role of deductive reasoning in establishing such proofs, and to develop the ability to construct such proofs
- to enhance those mathematical skills required for further studies in mathematics, the physical sciences and the technological sciences

Mathematics Outcomes

Objectives	Preliminary Outcomes	HSC Outcomes
Students will develop:	A student:	A student:
appreciation of the scope, usefulness, beauty and elegance of mathematics	P1 demonstrates confidence in using mathematics to obtain realistic solutions to problems	Hl seeks to apply mathematical techniques to problems in a wide range of practical contexts
the ability to reason in a broad range of mathematical contexts	P2 provides reasoning to support conclusions which are appropriate to the context	H2 constructs arguments to prove and justify results
skills in applying mathematical techniques to the solution of practical problems	P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques	H3 manipulates algebraic expressions involving logarithmic and exponential functions H4 expresses practical problems in mathematical terms based on simple given models H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

understanding of the key concepts of calculus and the ability to differentiate and integrate **a** range of functions	PS understands the concept of a function and the relationship between a function and its graph P6 relates the derivative of a function to the slope of its graph P7 determines the derivative of a function through routine application of the rules of differentiation	H6 uses the derivative to determine the features of the graph of a function H7 uses the features of a graph to deduce information about the derivative H8 uses techniques of integration to calculate areas and volumes
the ability to interpret and communicate mathematics in a variety of forms	P8 understands and uses the language and notation of calculus	H9 communicates using mathematical language, notation, diagrams and graphs

MATHEMATICS

The Mathematics syllabus has been divided into a Preliminary Course and HSC Course as follows:

Preliminary Course	HSC Course
Basic Arithmetic and algebra (1.1 - 1.4)	Coordinate methods in geometry (6.8)
Real functions (4.1 - 4.4)	Applications of Geometrical properties (2.5)
Trigonometric ratios (5.1 - 5 3)	Geometrical applications of differentiation (10.1 - 10.8)
Linear functions (6.1 - 6.5, 6.7)	Integration (11.1 - 11.4)
The quadratic polynomial and the parabola (9.1 - 9.5)	Trigonometric functions (including application of trigonometric ratios (13.1 13.6, 13.7)
Series (7.1 - 7.3)	Logarithmic and exponential functions (12.1- 12.5)
Plane geometry - geometrical properties (2.1 - 2.4)	Application of calculus to the physical world (14.1 -14.3)
Tangent to a curve and derivative of a function (8.1 - 8.9)	Probability (3.1 - 3.3)
	Series application (7.5)

Note: Numbers given are syllabus reference (1982 2 Unit and 3 Unit Mathematics syllabus)

Length of Course: Indicative class time of 120 hours of school study for the Preliminary course and of 120 hours of school study for the HSC Course

HSC Examination Format: A single written page of 3 hours duration, called the Mathematics/Mathematics Extension 1 (common) paper

MATHEMATICS

Mathematics Preliminary Course

NOTE: Each topic should be taught with reference to a calculator

1 BASIC ARITHMETIC AND ALGEBRA

a Arithmetic
  (i) four basic operations
  (ii) rational numbers-conversion of fractions to terminating and repeating decimals, percentages and vice versa
  (iii) powers and roots
  (iv) scientific notation, significant figures and approximations
  (v) quadratic surds: four operations, rationalising the denominator
b Absolute values
  (i) definition
  (ii) triangle inequality to be derived
  (iii) geometric interpretation
  (iv) simple graphs
c Substitution in and simplification of algebraic expressions
d Factorisation including sum and difference of two cubes
e Algebraic fractions
f Linear equations and inequalities
g Quadratic equations
h Simultaneous equations

2 REAL FUNCTIONS

a Dependent and independent variables, relations
b Definition of a function, notation, domain and range
c Graph of a function
  (i) bounded and unbounded domains and ranges
  (ii) continuous and discontinuous curves
  (iii) curves with simple symmetries
  (iv) curves with sharp corners
  (v) curves with asymptotes
d Odd and even functions and their symmetry
e Algebraic representation of geometrical relationships. Locus.
  (i) circle $(x - a)^2 + (y - b)^2 = r^2$
  (ii) points on either of the straight lines $y = \pm x$ ie $x^2 - y^2 = 0$
  (iii) perpendicular bisector of an interval
  (iv) parabola: $x^2 = 4ay$, $y = ax^2 + bx + c$
f Region and inequality
g Intersecting curves and regions

3 TRIGONOMETRIC RATIOS

a Trigonometric rations, using the unit circle. Reciprocal ratios
b The relation $\sin^2 A + \cos^2 A = 1$
c Trigonometric ratio of: -A, 90^0 - A, 180^0 - A, 360^0 - A.
d The exact ratios
e Bearings and angles of elevation and depression
f Sine and cosine rules (prove)
g Area of a triangle given 2 sides and the included angle (prove)
h Greatest and least values of trigonometric functions
i Solving trigonometric equations

4 LINEAR FUNCTIONS

a The linear function $y = mx + b$ and its graph
b Forms of the equation of a straight line
c Parallel and perpendicular lines
d Intersecting and coincident lines
e Equation of a line passing through the point of intersection of two given lines
f Distance between two points (derive)
g Perpendicular distance of a point from a line (derive)
h Midpoint of an interval (derive)

5 THE QUADRATIC POLYNOMIAL AND THE PARABOLA

a The quadratic polynomial $ax^2 + bx + c$
  (i) graphs of quadratic functions
  (ii) roots of quadratic equation by; graph
    factorisation
    completing the square
    formula
  (iii) quadratic inequalities
b General theory of quadratic equations
  (i) roots and coefficients
  (ii) discriminant
c Classification of quadratic expressions;
  (i) positive definite
  (ii) negative definite
  (iii) indefinite
d Identity of two quadratic expressions
e Equations reducible to quadratics
f Locus Problems
g Parabola defined as a locus. Vertex not at the origin
h Define focus, directrix, vertex, axis and focal length
i Greatest and least value when $x = \frac{-b}{2a}$

6 SERIES

a Review of index laws (Note: This is in the Year 12 HSC course but needs to be done before series and differentiation)
b Review of logarithm laws (HSC Course)
c Arithmetic series
(i) nth term (derive)
(ii) sum to n terms (derive)
d Geometric series
(i) nth term (derive)
(ii) sum to n terms (derive)
e Geometric series with a ration between -1 and 1
f The limit of x^n, as $n \to \infty$, for $x < 1$ (derive)
g Limiting sum

7 PLANE GEOMETRY

a Notation, symbols and conventions
b Definitions of special plane figures
c Angles at a point
d Angles formed by transversal to parallel lines
e Test for parallel lines
f Angle sums of triangles, quadrilaterals and general polygons
g Exterior angles properties
h Congruence of triangles. Tests for congruence
i Properties of special triangles and quadrilaterals
j Tests for special quadrilaterals
k Intercept properties of transversal to parallel lines
l Similarity of triangles. Tests for similarity
m Pythagoras' theorem and it's converse
n Area formulae
o Applications of the above properties to the solution of numerical exercises requiring one or more steps of reasoning

8 TANGENT TO A CURVE AND DERIVATIVE OF A FUNCTION

a Intuitive idea of continuity
b Continuous and discontinuous functions
c The concept of a limit of a function
d Properties of limits
e Definition of continuity
f Limits and continuity of sum, difference and product of two functions
g Definition of a secant
h Derive general expression for the gradient of the secant through two points
i Tangent as the limiting, position of the secant (illustrate by diagram)
j Formal definition of the derivative of f(x) and equivalent notations
k Equations of tangent and normal at a given point on the curve
l Differentiation of x^n (n a positive integer), from first principles
m Differentiation of $\sqrt{x}$ and x^{-1} from first principles
n Proofs of the following theorems where c is a constant, u = fg(x) and v = g(x)

(i) $$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

(ii) $$\frac{d}{dx}(u+v) = \frac{du}{dx} = \frac{dv}{dx}$$

(iii) $$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

(iv) $$\frac{d}{dx}F(x) = F'(u)\frac{du}{dx}$$

o Differentiation of x^n where n is rational
p The derivative of $\{f(x)\}^n$, and $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomials
q Numerous exercises to thoroughly drill the above concepts

MATHEMATICS

Mathematics HSC Course

9 COORDINATE METHODS IN GEOMETRY

Use of coordinate methods in solving geometrical problems restricted to problems with specified data.

10 APPLICAI'IONS OF GEOMETRICAL PROPERTIES

a Application of any property listed in Topic 7
b Problems should mainly have diagrams supplied although practice should be given in sketching a diagram from a given set of data
c Problems numerical or general, numerical stressed for Mathematics students. Theoretical problems requiring, one or more steps of reasoning
d A geometrical justification for each step required where appropriate

11 GEOMETRICAL APPLICATIONS OF DIFFERENTIATION

a Geometrical significance of the sign of *f '(x) :* f (x) increasing, decreasing, monotonic
b Stationary points and their nature
c Turning points
d Distinction between a local maximum and the greatest value of a function for a given domain
e Definition of the second derivative. Notation f "(x), $\frac{d^2y}{dx^2}$, y "
f Geometrical significance of the sign of the second derivative, ie. concavity
g Points of inflexion
h Sketching curves such as quadratics, cubics, higher polynomials and simple rational functions
  (i) symmetry
  (ii) very large positive and negative values of x
  (iii) points at which functions are defined
i Problems on maxima and minima
j Equations of tangents and normals to curves
k Primitive functions and their geometrical interpretation

12 INTEGRATION

a Notation of a limit
b "Intuitive" approach, ie 3,4..... n equal subdivisions of a < x < b and forming corresponding area sums to calculate the area enclosed by the x axis, x = a, x = b and the curve
c Integral notation
d Definite integrals
e The relation between the integral and the primitive function
f Calculation of areas, above and below the x axis
g Approximate methods:
  (i) Trapezoidal rule
  (ii) Simpson's rule
h Volumes of solids of revolution
i Volumes of solids of revolution. Derive results for cone and sphere
j Volumes of solids of revolution. Revolution about y axis

13 THE TRIGONOMETRIC FUNCTIONS

a Circular measure of angles
  (I) Definition of a radian
  (ii) Conversion from degrees to radians, vice versa
  (iii) Formulae involving radians
    α arc length (derive)
    β area of a sector
    γ area of a segment
b Graphs of circular functions, (including reciprocal functions), periodicity
c Graphical solutions of transcendental equations such as sin 3x = x
d Approximations to sin x, cos x and tan x when x is small
  (i) sin x < x < tan x, where x is acute
  (ii) $\lim_{x \to 0} \frac{\sin x}{x} = 1$
e Proof of sin (x + h) = sin.x cos h + sin.h cos x (not examinable)
f Derivatives of trigonometric functions (proof not examinable)
g Miscellaneous examples of the derivatives of trigonometric functions
h Integration of trigonometric functions and applications
i Extension to functions of the form asin (bx + c), etc

14 LOGARITHMIC AND EXPONENTIAL FUNCTIONS

a Review of definitions and properties of logarithms and exponents
b The exponential function and its graphs
c Calculation of e
d Differentiation of e^x and $e^{f(x)}$ and corresponding integrations
e The logarithmic function and its graph
f Inverse function and its graph
g Differentiation of log f(x) and corresponding integrations
h Application involving tangents, areas and volumes

NOTE: The notations ln x and log x for the natural logarithm $\log_e x$ should be known

15 APPLICATIONS OF CALCULUS TO THE PHYSICAL WORLD

a The rate of change of a quantity Q with respect to t defined as $\frac{dQ}{dt}$

b Sketches of Q and Q'(t*)* where possible

c Exponential growth and decay. (Mathematics students not expected to derive an equation from given information)

(i) Use direct substitution of $N = Ae^{kt}$ (A fixed) to demonstrate that is satisfies the equation for every A

(ii) growth rate (instantaneous rate of increase of populations)

(iii) initial population

(iv) population graphs

Emphasis on understanding the behaviour of the system

d Velocity and acceleration as time derivatives

e Average and instantaneous velocity

f Application involving;

(i) Determination of velocity and acceleration of a particle given its distance from a point as a function of time

(ii) Determination of the distance of a particle from a given point, given its acceleration of velocity as a function of time together with initial conditions

(iii) Determination of velocity and displacement given the acceleration and initial conditions i.e. integration

16 PROBABILITY

a Historical background

b Random experiments, equally likely outcomes, probability of a given result

c Finite sample space - simple, composite and mutually exclusive event, Venn diagrams

d Diagrammatic representation of sample spaces. Counting techniques

e $P(E) = \frac{n(E)}{n(S)}$, $0 < P(E) < 1$ Complementary results defined

f Addition theory or theorem of total probability

g Product theorem

h Dependent events - tree diagram (2 or 3 stage experiments)

i Independent events - product rule, tree diagrams

17 SERIES APPLICATIONS

a Review of arithmetic and geometric series

b Applications of arithmetic series

c Applications of geometric series

(i) compound interest

(ii) simplified hire purchase

(iii) repayment problems

d Application of recurring decimals

18 GENERAL REVISION

a Topic by topic

b Specimen papers

c Trial HSC papers

d H.SC papers

Marking Criteria

At the completion of your HSC course you will receive a Certificate of Achievement that indicates your level of achievement in each of your courses. Your final mark for each subject will place you in one of 6 bands (similar to your School Certificate), 6 being the highest and 2 being the lowest with only the minimum standard being achieved. Band 1 will be below the minimum standard needed to successfully complete the course.

To achieve a result in Bands 5 or 6 you need to consistently present solutions as described in the "Best Solutions" below. To achieve a result in bands 3 or 4 you need to consistently present solutions as described in the "Average Solutions" below. Presented solutions as described in the "Minimum Standard Solutions" below may give you a result in Band 2.

Your final mark is an average of your school assessment and your HSC examination mark. By the time you have completed the Preliminary Course you should have developed sufficient knowledge, skills and work habits to be able to consistently set out your solutions as described in "The Best Solutions" section below. As the school assessment is continuous throughout the HSC you will need to apply these points to all your work for the HSC course.

The Best Solutions should:

- Use a wide variety of problem solving strategies, successfully applying mathematical skills and processes to the most appropriate method.
- Clearly show understanding of the questions intent.
- Correctly use the language of mathematics including symbols, abbreviations, notation and conventions.
- Use graphs and diagrams to show understanding of the problem and as an aid in finding the solution.
- Demonstrate a good understanding of all the mathematical concepts involved.
- Recognise and fully explain each step involved in the solution in a clear logical sequence.
- Draw diagrams and graphs that are clearly labelled, reasonably sized and well executed using correct geometrical instruments (ruler, set square, pair of compasses).
- Give answers that are mathematically correct, expressed in the simplest form and with the correct units.
- State any formula used, clearly show the substitution and then evaluate.
- Clearly show the final answer, including writing the answer in a sentence when appropriate.
- Avoid using rounded values during the course of calculations and write down the full calculator answer before giving a rounded off answer.
- Leave incorrect and rough working still legible by only using a single line to cross out unwanted working in a solution.
- Show working spread out neatly, working down the page, not across, avoid using columns, ensure numbered parts are clearly indicated.
- Avoid irrelevant algebra and number crunching, or proving given information or giving essay answers when a sentence (or two) is enough.

The Average Solution should:

- Use a variety of problem solving strategies, applying mathematical skills and processes to the method used with only minor errors.
- Clearly show understanding of the questions intent.
- Correctly use the language of mathematics including symbols, abbreviations, notation and conventions in most parts of the solution.
- Use graphs and diagrams to show understanding of the problem and as an aid in finding the solution.
- Generally demonstrate understanding of the mathematical concepts involved.
- Explain each step involved in the solution in a clear logical sequence.
- Draw diagrams and graphs that are clearly labelled, reasonably sized and well executed using correct geometrical instruments (ruler, set square, pair of compasses).
- Generally give answers that are mathematically correct, expressed in the simplest form and with the correct units.
- State any formula used, clearly show the substitution and then evaluate.
- Clearly show the final answer, including writing the answer in a sentence when appropriate.
- Avoid using rounded values during the course of calculations and write down the full calculator answer before giving a rounded off answer.
- Leave incorrect and rough working still legible, using only a single line to cross out unwanted working in a solution.
- Show working spread out neatly, working down the page, not across, avoid using columns, ensure numbered parts are clearly indicated.

- Generally avoid irrelevant algebra and number crunching, or proving given information or giving essay answers when a sentence (or two) is enough.

The Minimum Standard Solutions should:

- Apply mathematical skills and processes to the method used with only minor errors.
- Correctly use some of the language of mathematics including symbols, abbreviations, notation and conventions.
- Use graphs and diagrams as an aid in finding the solution.
- Demonstrate some understanding of the mathematical concepts involved.
- Explain steps involved in the solution.
- Draw diagrams and graphs that are clearly labelled, reasonably sized and well executed using correct geometrical instruments (ruler, set square, pair of compasses).
- Generally give answers that are mathematically correct with correct units.
- State any formula used, clearly show the substitution and then evaluate.
- Clearly show the final answer, including writing the answer in a sentence when appropriate.
- Avoid using rounded values during the course of calculations and write down the full calculator answer before giving a rounded off answer.
- Leave incorrect and rough working still legible, using a single line to cross out unwanted working in a solution.
- Show working spread out neatly, working down the page, not across, avoid using columns, ensure numbered parts are clearly indicated.

DRAFT PERFORMANCE BANDS

MATHEMATICS

The typical performance in this band:

Band 6	Exhibits extensive knowledge and skills appropriate to the Mathematics course Uses sophisticated multi-step reasoning Integrates ideas of calculus with strong algebraic, deductive and modelling skills to successfully solve difficult problems Exhibits excellent problem solving skills Communicates effectively using appropriate mathematical language, notation, diagrams and graphs
Band 5	Exhibits sound knowledge and skills appropriate to the Mathematics course Uses multi-step logical reasoning in both numerical and theoretical contexts such as problems in calculus, geometry and probability Combines ideas of calculus with algebraic, deductive and modelling skills to successfully solve many difficult problems Exhibits a wide range of problem solving skills such as applications of series Communicates effectively using mathematical language, notation, diagrams, and graphs
Band 4	Exhibits the manipulative skills and knowledge base appropriate to the Mathematics course Uses logical reasoning in both numerical and theoretical contexts such as problems in calculus and geometry Identifies appropriate approaches to the solution of difficult problems Uses calculus and other methods to determine the features of, and to graph, a wide range of functions Successfully applies calculus and other appropriate ideas to model practical problems Communicates using mathematical language, notation, diagrams and graphs
Band 3	Consistently applies arithmetic and algebraic procedures correctly Applies geometrical reasoning in a numerical context Graphs functions such as $3\sin 2x$, $\log x$ and e^x Consistently applies rules of differentiation and basic integration correctly Uses calculus to determine the features of, and to graph, functions such as cubic polynomials Solves simple problems involving series
Band 2	Correctly applies arithmetic and basic algebraic procedures Recalls many of the formulae and algorithms appropriate to the Mathematics course, such as Simpson's rule, the sine rule, and the cosine rule Graphs simple functions such as linear functions, quadratics, $\sin x$ and $\cos x$ Finds derivatives of basic functions such as polynomials, $\sin x$ and e^x Uses the rules of differentiation such as the product rule Solves numerical problems involving the geometry of triangles
Band 1	